

2012 NUMERICAL ANALYSIS EXAMINATION

Problem 1. Let f be a three times continuously differentiable function on $[-1, 1]$. Interpolate the function by a quadratic polynomial $p_2(x)$ using the support points: $(-1, f(-1))$, $(0, f(0))$, $(1, f(1))$.

Problem 2. From **Problem 1**, what is the maximal absolute interpolation error

$$\max_{x \in [-1, 1]} |f(x) - p_2(x)|.$$

Make the table for the divided differences at the nodes.

Problem 3. Consider the integral

$$\int_0^1 f(x) \, dx.$$

- (1) Derive the Trapezoidal rule for the integral.
- (2) Give the error representation for the quadrature rule.
- (3) Compute the error for the function $f(x) = x^2$ on $[0, 1]$.