

B.Sc. Examination — Introduction to Differential Geometry

Semester I of 2012, Mathematics, POSTECH

[1] Compute the curvature and the torsion of the space curve (in the three dimensional Euclidean space \mathbf{R}^3 of the helix

$$\alpha(t) = (3 \cos t, 2t, 3 \sin t).$$

[2] Compute the Gauss curvature of the helicoid $f(u, v) = (v \cos u, 5v, -v \sin u)$ at the point $(u, v) = (\pi, 1)$.

[3] Consider the unit sphere S in \mathbf{R}^3 centered at the origin. Let P be the plane defined by $x + y + z = 1$, which passes through the points $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$. Then the intersection $P \cap S$ is a curve (a circle!); call it γ . Explain why this γ cannot be a geodesic on S , in whichever ways one parametrizes it.