EXAM
Discrete Mathematics
May 2012

(1) Prove the identity

$$\sum \frac{1}{n_1 \cdot n_2 \cdots n_k} = n$$

where the sum runs over all nonempty subsets $\{n_1, n_2, \ldots, n_k\}$ of $\{1, 2, \ldots, n\}$.

(2) The Fibonacci sequence f_n is given by $f_1 = 1$, $f_2 = 2$, and

$$f_n = f_{n-1} + f_{n-2}$$

for $n \geq 3$. Find an explicit formula for f_n.

(3) Let A be the adjacency matrix of a simple graph. Show that for any integer $n \geq 1$, the (i, j) entry of A^n is equal to the number of paths of length n from vertex i to vertex j. (Hint: Use induction on n.)