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최영주

제 1 절 3 Problems

(1) (10 pt) Find the inverse of

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 1 & 0 & 2 \end{pmatrix}.$$

(Hint Apply Gauss-Jordan elimination to $[A|I]$.)

(2) Consider the set C of the columns of the below matrix

$$\begin{pmatrix} 1 & -2 & -1 & 0 \\ 4 & 2 & 6 & 8 \\ 2 & -1 & 1 & 3 \end{pmatrix}$$

Is this set C linearly dependent over the real $\mathbb{R}$? Justify your answer.

(3) Let $w_1 = (1, 0), w_2 = (2, -1), w_3 = (4, 3)$ be three vectors in $\mathbb{R}^2$. Let $\alpha = \{e_1, e_2, e_3\}$ be the standard basis for the 3−space $\mathbb{R}^3$, and let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by

$$T(e_1) = w_1, T(e_2) = w_2, T(e_3) = w_3.$$

Find a formula for $T(x_1, x_2, x_3)$, and then use it to compute $T(2, -3, 5)$.

제 2 절 응용 선형대수 졸업시험 Solution

(1)

$$[A|I] = \begin{pmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 2 & 3 & 5 & | & 0 & 1 & 0 \\ 1 & 0 & 2 & | & 0 & 0 & 1 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & -1 & -1 & | & -2 & 1 & 0 \\ 0 & -2 & -1 & | & -1 & 0 & 1 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & 2 & -1 & 0 \\ 0 & -2 & -1 & | & -1 & 0 & 1 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & 2 & -1 & 0 \\ 0 & 0 & 1 & | & 3 & -2 & 1 \end{pmatrix}$$

Now do back substitution:

$$\rightarrow \begin{pmatrix} 1 & 2 & 0 & | & -8 & 6 & -3 \\ 0 & 1 & 0 & | & -1 & 1 & -1 \\ 0 & 0 & 1 & | & 3 & -2 & 1 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & & 0 & | & -6 & 4 & -1 \\ 0 & 1 & 0 & | & -1 & 1 & -1 \\ 0 & 0 & 1 & | & 3 & -2 & 1 \end{pmatrix}$$

So

$$A^{-1} = \begin{pmatrix} 1 & 0 & 0 & | & -6 & 4 & -1 \\ 0 & 1 & 0 & | & -1 & 1 & -1 \\ 0 & 0 & 1 & | & 3 & -2 & 1 \end{pmatrix}$$

(2) No! The third column is the sum of the first and the second

(3) For $X = (x_1, x_2, x_3) = x_1e_1 + x_2e_2 + x_3e_3 \in \mathbb{R}^3$,

$$T(X) = \sum_{i=1}^{3} x_i T(e_i) = \sum_{i=1}^{3} x_i w_i$$

$$= x_1(1,0) + x_2(2,-1) + x_3(4,3)$$

$$(x_1 + 2x_2 + 4x_3, -x_2 + 3x_3)$$

Thus, $T(2,-3,5) = (16,18)$