

## Complex variables

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1. Find the first two terms of the Laurent series of the function  $f(z) = \tan z$  about  $z = \pi/2$ .
2. Show that there is exactly one root inside the contour  $C_1 : |z| = 1$ , for

$$h(z) = e^z - 4z - 1.$$

3. Use residues to show that

$$\int_{-\infty}^{\infty} \frac{x^2}{x^4 + 1} dx = \frac{\pi}{\sqrt{2}}.$$