PMI
Pohang Mathematics Institute

**Department of Mathematics**
**Pohang University of Science & Technology**
San31 Hyoja-Dong Nam-Gu, Pohang 790-784, Korea
Tel.+82-54-279-5510, Fax. +82-54-279-2799

# Five Gold Button Math Competition

May 2011

Let $\phi \in C^1(\mathbb{R}, \mathbb{R}^m)$ be such that $\phi$ and $\phi'$ are square integrable and

$$\{t \,|\, \phi(t) = 0\} \subseteq \{t \,|\, \phi'(t) = 0\}$$
$$|\{t \,|\, \phi(t) = 0\}| = n \in \mathbb{N}.$$

Prove that the inequality holds:

$$\int_{\mathbb{R}} |\phi|'^2(t)dt \leq \int_{\mathbb{R}} |\phi'(t)|^2 dt.$$

Prove also that if $\int_{\mathbb{R}} |\phi|'^2(t)dt = \int_{\mathbb{R}} |\phi'(t)|^2 dt$, then there are $n+1$ points

$$\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{n+1} \in S^{m-1} = \{\mathbf{x} \in \mathbb{R}^m \,|\, |\mathbf{x}| = 1\}$$

such that for each $t \in \mathbb{R}$, $\phi(t) = \mathbf{x}_i|\phi(t)|$ for some $i$.

Sponsored by
Department of Mathematics
Pohang Mathematics Institute
and University Development Committee
of POSTECH
Correspondence: five-goldpostech.ac.kr