Quantum probability theory provides a framework of extending the measure-theoretical (Kolmogorovian) probability theory. The idea traces back to von Neumann (1932), who, aiming at the mathematical foundation for the statistical questions in quantum mechanics, initiated a parallel theory by making a self-adjoint operator and a trace play the roles of a random variable and a probability measure, respectively. During the last 25 years quantum probability theory has developed considerably with wide applications. From a functional analytic viewpoint, one of the main themes of quantum probability theory is to investigate spectral properties of non-commuting operators from the viewpoint of “probability.”

It may well sound strange that there are several different concepts of independence, contrary to the classical (Kolmogorovian) probability theory. In the quantum probability, we have so far established many different concepts of independence based upon non-commutativity. In these lectures we discuss four basic concepts of independence and the corresponding central limit theorems. These results are applied to (asymptotic) spectral analysis of large (or growing) graphs.

For more detailed and relevant information, see

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